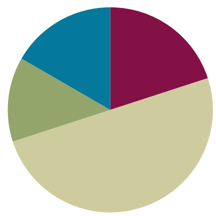
Lesson 3

Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Multiply Mentally **4.OA.4** (4 minutes)
- Repeated Addition as Multiplication **4.OA.4** (4 minutes)
- Add Fractions **4.NF.3** (4 minutes)

Multiply Mentally (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 3 content.

T: (Write 34 × 2 = ____.) Say the multiplication sentence.

S: 34 × 2 = 68.

T: (Write 34 × 2 = 68. Below it, write 34 × 20 = ____.) Say the multiplication sentence.

S: 34 × 20 = 680.

T: (Write 34 × 20 = 680. Below it, write 34 × 22 = ____.) On your personal white board, solve 34 × 22.

S: (Write 34 × 22 = 748.)

Continue with the following possible sequence: 23 × 3, 23 × 20, and 23 × 23; and 12 × 4, 12 × 30, and 12 × 34.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Scaffold the Multiply Mentally fluency activity for students working below grade level and others having difficulty. Clarify that (34 × 2) + (34 × 20) is the same as 34 × 22, and so on. Ask students, "Why is this true?"

Repeated Addition as Multiplication (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 3 content.

T: (Write 2 + 2 + 2 = ____.) Say the addition sentence.

S: 2 + 2 + 2 = 6.

T: (Write 2 + 2 + 2 = 6. Beneath it, write __ × 2 = 6.) On your personal white board, fill in the unknown factor.

S: (Write 3 × 2 = 6.)

T: (Write 3 × 2 = 6. To the right, write 30 + 30 + 30 = ____.) Say the addition sentence.

2 + 2 + 2 = 6	30 + 30 + 30 = 90	32 + 32 + 32 = 96
3 × 2 = 6	3 × 30 = 90	3 × 32 = 96

S: 30 + 30 + 30 = 90.

T: (Write 30 + 30 + 30 = 90. Beneath it, write ____ × 30 = 90.) Fill in the unknown factor.

S: (Write 3 × 30 = 90.)

T: (Write 3 × 30 = 90. To the right, write 32 + 32 + 32 = ____.) On your board, write the repeated addition sentence. Then, beneath it, write a multiplication sentence to reflect the addition sentence.

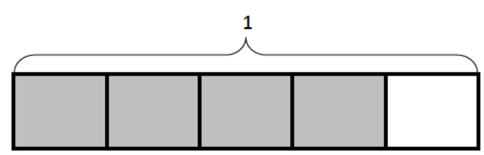
S: (Write 32 + 32 + 32 = 96. Beneath it, write 3 × 32 = 96.)

Continue with the following possible sequence: 1 + 1 + 1 + 1, 4 × 1; 20 + 20 + 20 + 20, 4 × 20; 21 + 21 + 21 + 21, 4 × 21; and 23 + 23 + 23, 3 × 23.

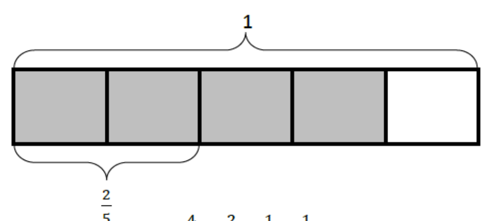
Add Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 2.



$\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$



$\frac{4}{5} = \frac{2}{5} + \frac{1}{5} + \frac{1}{5}$

T: (Write $\frac{4}{5}$.) Say the fraction.

S: 4 fifths.

T: On your personal white board, draw a tape diagram representing 4 fifths.

S: (Draw a tape diagram partitioned into 5 equal units. Shade 4 units.)

T: (Write $\frac{4}{5} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$.) Write $\frac{4}{5}$ as the sum of unit fractions.

S: (Write $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.)

T: (Write $\frac{4}{5} = \frac{2}{5} + \frac{\quad}{5} + \frac{\quad}{5}$.) Bracket 2 fifths on your diagram, and complete this number sentence.

S: (Group $\frac{2}{5}$ on the diagram. Write $\frac{4}{5} = \frac{2}{5} + \frac{1}{5} + \frac{1}{5}$.)

T: (Write $\frac{4}{5} = \frac{\quad}{5} + \frac{\quad}{5}$.) Bracket fifths again on your diagram, and write a number sentence to match. There's more than one correct answer.

S: (Group fifths on the diagram. Write $\frac{4}{5} = \frac{2}{5} + \frac{2}{5}$, $\frac{4}{5} = \frac{3}{5} + \frac{1}{5}$, or $\frac{4}{5} = \frac{1}{5} + \frac{3}{5}$.)

Continue with the following possible sequence: $\frac{5}{6} = \frac{\quad}{6} + \frac{\quad}{6} + \frac{\quad}{6} + \frac{\quad}{6} + \frac{\quad}{6}$, $\frac{5}{6} = \frac{\quad}{6} + \frac{3}{6} + \frac{\quad}{6}$, and $\frac{5}{6} = \frac{\quad}{6} + \frac{\quad}{6} + \frac{\quad}{6}$.

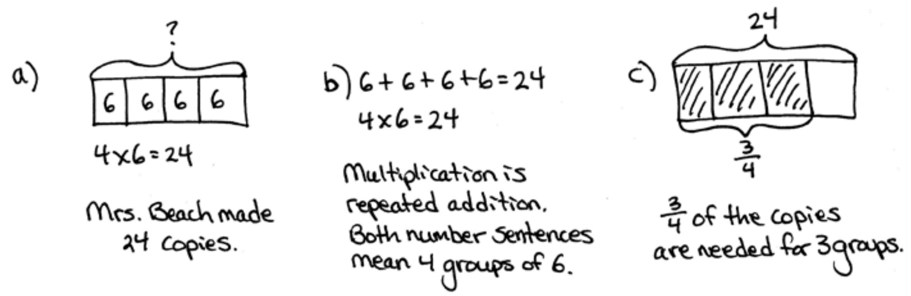
Application Problem (8 minutes)

Mrs. Beach prepared copies for 4 reading groups. She made 6 copies for each group. How many copies did Mrs. Beach make?

a. Draw a tape diagram.

b. Write both an addition and a multiplication sentence to solve. Discuss with a partner why you are able to add or multiply to solve this problem.

c. What fraction of the copies is needed for 3 groups? To show that, shade the tape diagram.



Note: This Application Problem builds from Grade 3 knowledge of interpreting products of whole numbers. This Application Problem bridges to today's lesson where students come to understand that a non-unit fraction can be decomposed and represented as a whole number times a unit fraction.

Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1: Express a non-unit fraction less than 1 as a whole number times a unit fraction using a tape diagram.

T: Look back at the tape diagram that we drew in the Application Problem. What fraction is represented by the shaded part?

S: $\frac{3}{4}$.

T: Say $\frac{3}{4}$ decomposed as the sum of unit fractions.

S: $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

T: How many fourths are there in $\frac{3}{4}$?

S: 3.

T: We know this because we count 1 fourth 3 times. Discuss with a partner. How might we express this using multiplication?

S: We have 3 fourths. That's $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ or three groups of 1 fourth. Could we multiply $3 \times \frac{1}{4}$?

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$\frac{3}{4} = 3 \times \frac{1}{4}$$

T: Yes! If we want to add the same fraction of a certain amount many times, instead of adding, we can multiply. Just like you multiplied 6 copies 4 times, we can multiply 1 fourth 3 times. What is 3 copies of $\frac{1}{4}$?

S: It's $\frac{3}{4}$. My tape diagram proves it!

Repeat with $\frac{2}{3}$ and $\frac{7}{8}$. Instruct students to draw a tape diagram to represent each fraction (as on the previous page), to shade the given number of parts. Then, direct students to write an addition number sentence and a multiplication number sentence.

Problem 2: Determine the non-unit fraction greater than 1 that is represented by a tape diagram, and then write the fraction as a whole number times a unit fraction.

T: (Project the tape diagram of $\frac{10}{8}$ as shown below.) What fractional unit does the tape diagram show?

S: It shows tenths! → It shows eighths!

T: We first must identify 1. It's bracketed here. (Point.) How many units is 1 partitioned into?

S: 8.

T: The bracketed portion of the tape diagram shows 8 fractional units. What is the total number of eighths?

$\frac{10}{8} = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$

$\frac{10}{8} = 10 \times \frac{1}{8}$

S: 10.

T: What is the fraction?

S: 10 eighths.

T: Say this as an addition number sentence. Use your fingers to keep track of the number of units as you say them.

S: $\frac{10}{8} = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$.

T: As a multiplication number sentence?

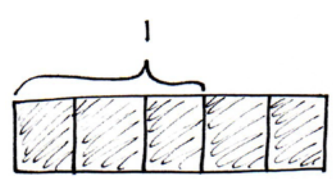
S: $\frac{10}{8} = 10 \times \frac{1}{8}$.

T: What are the advantages of multiplying fractions instead of adding?

S: It's easier to write. → It's faster. → It's more efficient.

Problem 3: Express a non-unit fraction greater than 1 as a whole number times a unit fraction using a tape diagram.

T: Let's put parentheses around 8 eighths so that we can see 10 eighths can also be written to show 1 and 2 more eighths. (Write $\frac{10}{8} = \left(8 \times \frac{1}{8}\right) + \left(2 \times \frac{1}{8}\right)$.)



$\frac{5}{3} = 5 \times \frac{1}{3}$

$\frac{5}{3} = \left(3 \times \frac{1}{3}\right) + \left(2 \times \frac{1}{3}\right)$

T: Discuss with your partner how to draw a tape diagram to show 5 thirds.

S: I can draw one unit at a time. The units are thirds, so I'll draw five small rectangles together. → I know 5 thirds is greater than 1, so I'll draw 1. That's 3 thirds. So, then I can draw another 1. I'll just shade 5 parts. → I will draw a long rectangle and break it into 5 equal parts. Each part represents 1 third. I'll bracket 3 thirds to show 1.

T: How can we express $\frac{5}{3}$ as a multiplication expression?

S: We have five thirds. That's $5 \times \frac{1}{3}$.

T: Is there another way we can express $\frac{5}{3}$ using multiplication?

S: Can we express the 1 as $3 \times \frac{1}{3}$ and then add $2 \times \frac{1}{3}$?

T: Yes! We can use multiplication and addition to decompose fractions.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Offer an alternative to Problem 2 on the Problem Set for students working above grade level. Challenge students to compose a word problem of their own to match one or more of the tape diagrams they construct for Problem 2. Always offer challenges and extensions to learners as alternatives, rather than additional *busy* work.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

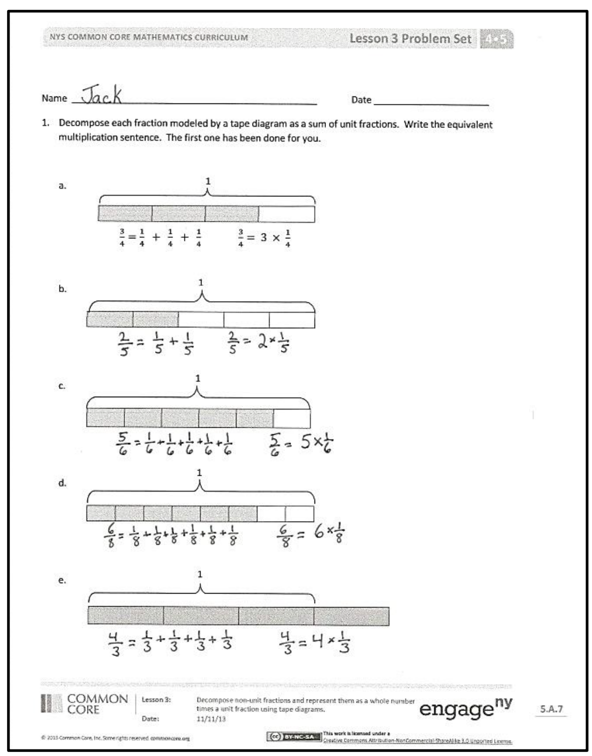
Lesson Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In all of the problems, why do we need to label 1 on our tape diagrams? What would happen if we did not label 1?



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Problem Set 4•5

Name Jack Date

1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence. The first one has been done for you.

a. $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ $\frac{3}{4} = 3 \times \frac{1}{4}$

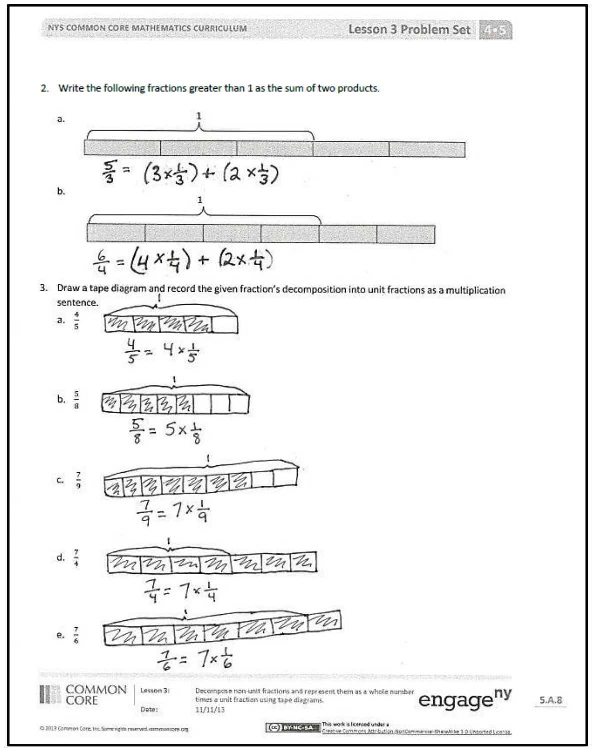
b. $\frac{2}{5} = \frac{1}{5} + \frac{1}{5}$ $\frac{2}{5} = 2 \times \frac{1}{5}$

c. $\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ $\frac{5}{6} = 5 \times \frac{1}{6}$

d. $\frac{6}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ $\frac{6}{8} = 6 \times \frac{1}{8}$

e. $\frac{4}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ $\frac{4}{3} = 4 \times \frac{1}{3}$

- What is an advantage to representing the fractions using multiplication?
- What is similar in Problems 3(c), 3(d), and 3(e)? Which fractions are greater than 1? Which is less than 1?
- Are you surprised to see multiplication sentences with products less than 1? Why?
- In our lesson, when we expressed $\frac{5}{3}$ as $\left(3 \times \frac{1}{3}\right) + \left(2 \times \frac{1}{3}\right)$, what property were we using?
- Consider the work we did in Lessons 1 and 2 where we decomposed a tape diagram multiple ways. Can we now rewrite those number sentences using addition and multiplication? Try it with this tape diagram (as shown below).



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Problem Set 4•5

2. Write the following fractions greater than 1 as the sum of two products.

a. 1

$\frac{5}{3} = (3 \times \frac{1}{3}) + (2 \times \frac{1}{3})$

b. 1

$\frac{6}{4} = (4 \times \frac{1}{4}) + (2 \times \frac{1}{4})$

3. Draw a tape diagram and record the given fraction's decomposition into unit fractions as a multiplication sentence.

a. $\frac{4}{5}$ $\frac{4}{5} = 4 \times \frac{1}{5}$

b. $\frac{5}{8}$ $\frac{5}{8} = 5 \times \frac{1}{8}$

c. $\frac{7}{9}$ $\frac{7}{9} = 7 \times \frac{1}{9}$

d. $\frac{7}{4}$ $\frac{7}{4} = 7 \times \frac{1}{4}$

e. $\frac{7}{6}$ $\frac{7}{6} = 7 \times \frac{1}{6}$

COMMON CORE Lesson 3: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams. Date: 11/11/13 engage^ny 5.A.8

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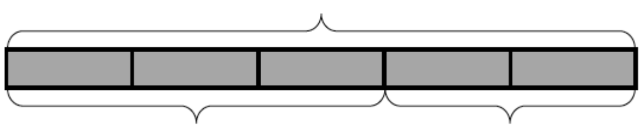


$\frac{3}{5}$ $\frac{2}{5}$

$$\frac{5}{5} = \frac{3}{5} + \frac{2}{5}$$

$$\frac{5}{5} = \left(3 \times \frac{1}{5}\right) + \left(2 \times \frac{1}{5}\right)$$

- How is multiplying fractions like multiplying whole numbers?
- How did the Application Problem connect to today's lesson?

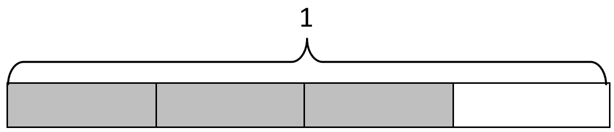
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ____________________

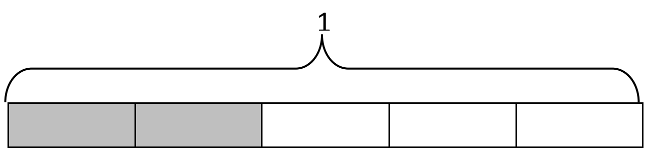
1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence. The first one has been done for you.

a.

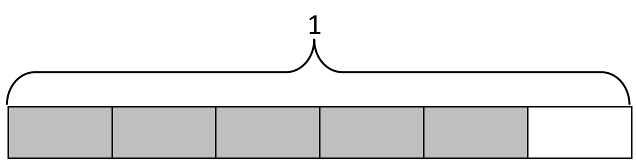


$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ $\frac{3}{4} = 3 \times \frac{1}{4}$

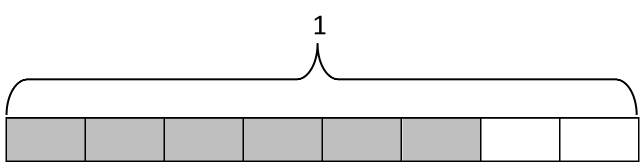
b.



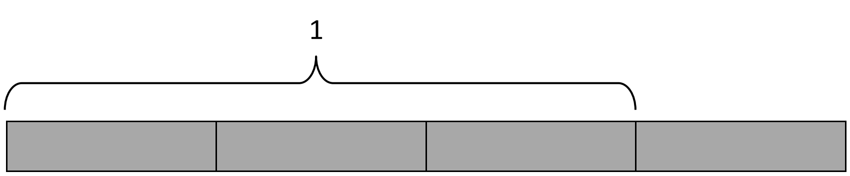
c.



d.

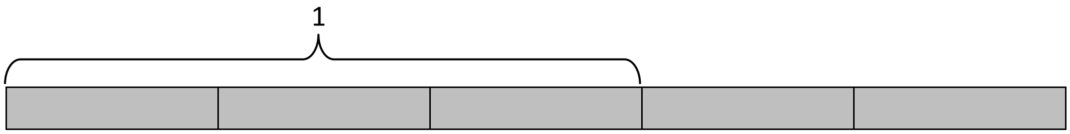


e.

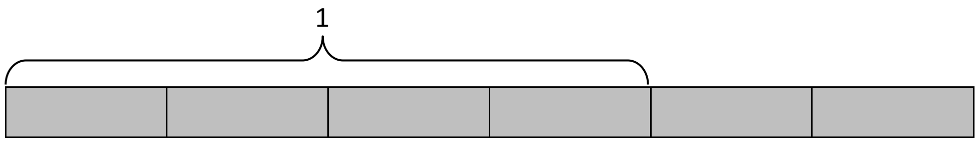


2. Write the following fractions greater than 1 as the sum of two products.

a.



b.



3. Draw a tape diagram, and record the given fraction's decomposition into unit fractions as a multiplication sentence.

a. $\frac{4}{5}$

b. $\frac{5}{8}$

c. $\frac{7}{9}$

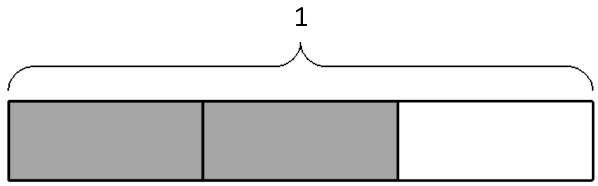
d. $\frac{7}{4}$

e. $\frac{7}{6}$

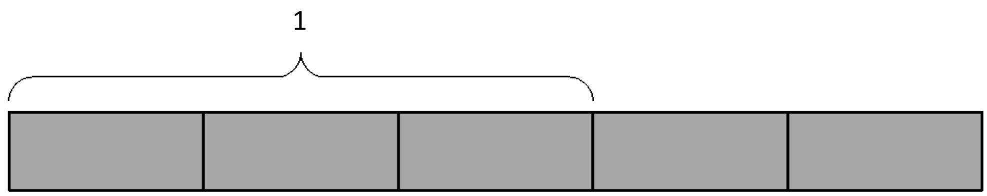
Name ____ Date ______________________

1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence.

   a.



   b.

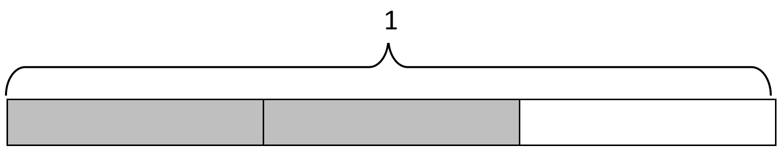


2. Draw a tape diagram, and record the given fraction's decomposition into unit fractions as a multiplication sentence.

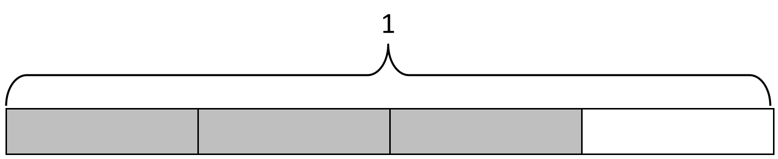
$$\frac{6}{9}$$

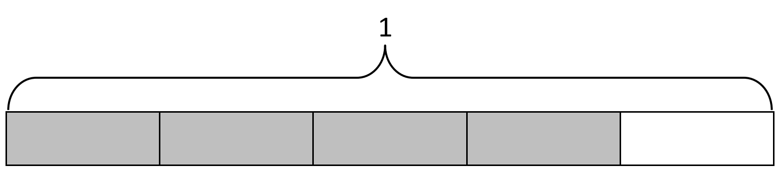
Name ____ Date ______________________

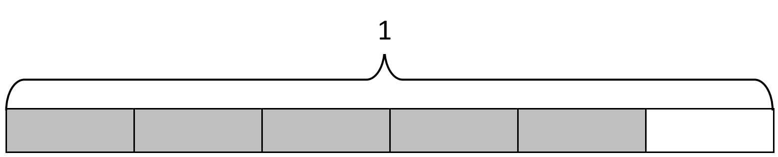
1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence. The first one has been done for you.

a.


$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$ $\frac{2}{3} = 2 \times \frac{1}{3}$

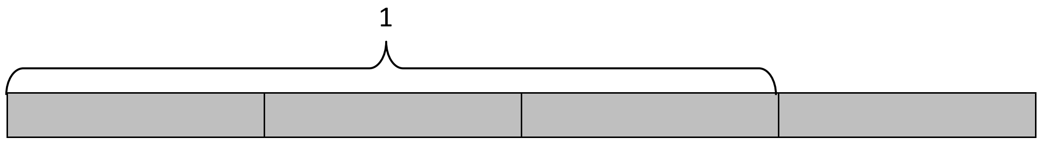
b.


c.


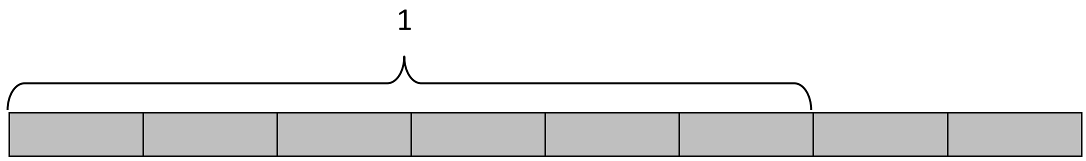
d.


2. Write the following fractions greater than 1 as the sum of two products.

   a.



   b.



3. Draw a tape diagram, and record the given fraction's decomposition into unit fractions as a multiplication sentence.

   a. $\frac{3}{5}$

   b. $\frac{3}{8}$

   c. $\frac{5}{9}$

   d. $\frac{8}{5}$

   e. $\frac{12}{4}$